On the question of the flatness of the Universe

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This paper examines the relationship between the entropy of the cosmological horizon and the flatness of the universe. It demonstrates the necessity of the universe being uncurved. It is shown that in the case of a flat universe expanding, entropy production is maximal compared to the expansion of a curved universe.

I. INTRODUCTION

Over the past 50 years, cosmology has become an important and precise part of modern theoretical physics thanks to the development of high-tech observational instruments and statistical methods for data processing, the entry into near-Earth orbit, the expansion of the observational range, and a number of discoveries. In 1965, Penzias and Wilson [1] the discovery of the cosmic microwave background (CMB) radiation. Measuring its precise parameters and anisotropy led to the conclusion that we live in a homogeneous and isotropic (to five decimal places) universe, greatly reducing the arbitrariness of possible descriptive models. Measurements of the Hubble parameter and several other experiments yielded results that allow us to assert that our universe is currently flat, and it is highly likely that this characteristic does not change over time[2]. The ΛCDM model assumes that the evolution of the Universe began with the decay of the vacuum state of a certain field, leading to the inflationary stage of expansion, and then, through several phase transitions, to the picture we observe. After the inflationary stage, the field decays and "ordinary"matter is born and heated up. The subsequent evolution of the Universe is determined by the dynamics of various components of matter: baryonic matter, radiation, dark matter (DM), and, separately, dark energy (DE), which, according to the majority of the scientific community, is a manifestation of the action of the physical vacuum on "ordinary"matter[3]. It should be noted that, despite the increasing accuracy of observations, the question of the curvature of space in the Universe remains open. Available data do not allow for a definitive conclusion about a flat Universe and allow for the presence of, albeit small, intrinsic curvature. This requires that, when rigorously considering questions related to the evolution of the Universe, the contribution of intrinsic curvature to its dynamics be taken into account.

The range of questions related to the thermodynamics of the Universe is broad. The relationship between entropy and gravity has been presented in a number of papers since the 1970s. Bekenstein [5], based on Hawking's result on the non-decreasing absolute event horizon, identified the entropy of a black hole with the surface area of this horizon. Subsequently, Zeldovich [6] and, later, Hawking [7]–[9] showed that a black hole should radiate like an absolutely black body, possessing a corresponding temperature T, proportional to the surface gravity of the black hole. The enormous entropy of black holes, according to Bekenstein [10, 11], arises because the state of a black hole provides no information about the growth of the specific system that formed it. The entropy of a black hole is considered to be the maximum possible entropy for objects of a given mass. Around the same time, Unruh published a paper, which demonstrated the existence of thermal radiation with a corresponding temperature, T, proportional to the gravitational acceleration.

In addition, a number of studies have substantiated and developed a unified approach to considering cosmological horizons by analogy with black holes. They demonstrated the identity of the description of the black hole event horizon and the cosmological horizon as "causal boundaries."The authors proved that the cosmological horizon has a temperature and entropy, just like a black hole. They showed that an observer in such a universe would detect thermal radiation emanating from the horizon, and the entropy formula $S = A/4$, where S is the entropy of the horizon and A is its area, remains valid. This leads to the fact that in a Universe with horizons, for the generalized second law of thermodynamics to be fulfilled, the entropy of the cosmological horizon must exist and obey the area law, otherwise the information (and entropy) going beyond the horizon would simply disappear from the Universe.

In addition to the above, it can be added that a number of works have appeared deriving gravity from entropy and vice versa, for example, [15]–[19]. Despite convincing arguments on both sides, the question of which is "primary"remains open. Nevertheless, it is clear that there is a connection between entropy and gravity, and this connection is fundamental. In this paper, we examine the interdependence of entropy, spatial curvature, and the density of the Universe.

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II. COSMOLOGICAL HORIZON, ITS ENTROPY AND THE DENSITY OF MATTER

In modern cosmology there is a problem of the flatness of the Universe. Based on observational data, it can be said that the Universe is flat with a high accuracy ($\sim 10^{-5}$). And this relation is preserved, at least since the epoch of recombination, when radiation separated from matter ($z \sim 1100$). The critical density $\rho_{crit}(t)$ corresponds to the flat state of the Universe. At the current moment of evolution, the numerical value $\rho_{crit}(t)$ is $\sim 10^{-30}$ g/cm^3. Taking into account the dynamics of density fluctuations, in the first seconds of expansion the deviation of the density from the critical $\left|\frac{\rho}{\rho_{crit}} - 1\right|$ should have been $\sim 10^{-59}$[?]. We are trying to understand and explain the mechanism of this "fine-tuning"of the Universe's parameters. Let's start with the formula for the entropy of the cosmological horizon (hereinafter referred to as the horizon). As mentioned above, this formula coincides with the Bechstein formula for the entropy of the S(t) black hole

$$S(t) = \frac{k_B c^3}{4G\hbar} A(t) \tag{1}$$

where k_B is the Boltzmann constant, c is the speed of light in vacuum, G is the gravitational constant, $\hbar$ is the reduced Planck constant, $A(t)$ is the area of the horizon. In turn, the area depends on the radius of the horizon $R_h(t)$ as

$$A(t) = 4\pi R_h(t)^2 \tag{2}$$

In our calculations, we assume that the Universe is not flat. Therefore, in the formula for $R_h(t)$ we explicitly take into account the spatial curvature $K(t) = \frac{k_0}{a(t)^2}$, where k_0 is the dimensionless curvature constant, $a(t)$ is the scale factor,

$$R_h(t) = \frac{c}{\sqrt{H(t)^2 + c^2 K(t)}} \tag{3}$$

Here $H(t)$ is the Hubble parameter. This gives us a formula for the horizon area

$$A(t) = 4\pi R_h(t)^2 = \frac{4\pi c^2}{H(t)^2 + c^2 K(t)} \tag{4}$$

Finally, in the general case of a Universe of arbitrary curvature, for the entropy of the horizon we obtain

$$S(t) = \frac{\pi k_B c^5}{G\hbar} \cdot \frac{1}{H(t)^2 + c^2 K(t)} \tag{5}$$

Now let's turn to Einstein's equations. In the FRLW metric

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[\frac{dr^2}{1 - k_0 r^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{6}$$

Einstein's first equation can be written as

$$H(t)^2 = \frac{8\pi G}{3}\rho(t) - c^2 K(t) \tag{7}$$

$\rho(t)$ takes into account *all* types of matter and energy: baryon and dark matter, radiation and DE. The dependence on curvature is expressed explicitly and taken out as a separate term, therefore $\rho(t)$ will be equal to $\rho_{crit}(t)$. This becomes clear if we equate the second term on the right-hand side of (7) to 0 - we obtain an equation for a *flat* Universe. We rewrite (7) taking into account the above

$$H(t)^2 = \frac{8\pi G}{3}\rho_{crit}(t) - c^2 K(t) \tag{8}$$

Substitute the value $H(t)^2$ from (8) into (5)

$$S(t) = \frac{3k_B c^5}{8G^2\hbar} \cdot \frac{1}{\rho_{crit}(t)} \tag{9}$$

As we can see, the entropy has ceased to depend on curvature and has come to correspond to the entropy of a flat Universe. That is, the entropy of the horizon changes without being affected by the presence or absence of spatial curvature of the Universe. It should be added that since the entropy of the horizon is invariant, the result will not depend on the choice of reference frame. Let's perform one more transformation. Multiply both parts (9) by $\rho_{crit}(t)$

$$\Psi = S(t)\cdot \rho_{crit}(t) = \frac{3k_B c^5}{8G^2\hbar} \tag{10}$$

The resulting value of Ψ is a constant and is expressed only in terms of fundamental constants. This value is approximately $2.66885\cdot 10^{96}$ (in units of $k_B \frac{kg}{m^3}$).

Let's express this quantity using Planck quantities. By simple transformations, we obtain the following formula

$$\Psi = \frac{3k_b c^5}{8G^2\hbar} = \frac{3}{8}\frac{k_B M_{Pl}}{L_{Pl}^3}, \tag{11}$$

where M_{Pl} and L_{Pl} are the Planck mass and length, respectively. The existence of such a constant demonstrates the deep connection between horizon thermodynamics and gravitational dynamics, which was proposed in the work [21]. The structure of Ψ and its constancy require that the Universe remain flat from the very beginning and subsequently, during its evolution. The mechanism for implementing this requirement will be discussed in the second chapter of this work. This also shows that the energy density in the Universe always corresponds to the information capacity of its horizon. This correspondence was set at the origin of the Universe (most likely after inflation) and has not changed since then. The value of Ψ was determined at the same moment as the selection of the fundamental constants of our Universe.

A. Model HDE

The paper [22] describes a model of holographic dark energy (HDE). This model applies the holographic principle to explain the accelerated expansion of the Universe. The basic idea is that the vacuum energy density ρ_Λ is determined not by quantum fluctuations at Planck scales (which yields an error of 120 orders of magnitude), but by the maximum entropy that a region of space can accommodate. This leads to ρ_Λ ceasing to be a constant value and changing its equation of state. In this part of the section, we will use Planck units.

According to the holographic principle, the entropy S of a region with characteristic size L is limited by the area of its boundary (similar to (2))

$$S \leq \frac{\pi L^2}{L_{Pl}^2}. \tag{12}$$

The paper argues that in quantum field theory, since the short-range confinement is coupled to the long-range confinement due to the limit imposed by black hole formation, namely, if ρ_Λ is the quantum zero-point energy density caused by the short-range confinement, then the total energy in a region of size L must not exceed the mass of a black hole of the same size, hence $L^3\rho_\Lambda \leq LM_{Pl}^2$. The largest allowable L is the one that satisfies this inequality, thus,

$$\rho_\Lambda = 3c^2 M_{Pl}^2 L^{-2}, \tag{13}$$

where c is the dimensionless constant of the model (of order 1), M_{Pl} is the reduced Planck mass. If we take the future horizon as L and define the effective mass within this horizon as $M_\Lambda = \rho_\Lambda V$, where V is the volume within this horizon, then, due to the linear dependence of M_Λ on L, the increase in mass exactly compensates for the decrease in the volumetric entropy density $s_v (s_v = \frac{S}{V})$, i.e.

$$s_v M_\Lambda = const \tag{14}$$

It is easy to verify that this constant is identical in magnitude to the previously obtained Ψ.

The paper also presents calculations of ρ_Λ demonstrating that this is a dynamical quantity, not a constant. In 2024, the results of processing observational data from the $DESI$ experiment, which studied baryon acoustic oscillations, were published. [23] The results of this paper show that the HDE model is at least consistent with these data, and, given reasonable assumptions about its parameters, is confirmed by them.

It should be noted that the results from [22] and the results obtained above were obtained using different approaches: consideration of the parameters of black holes (BH) in [22] and the dynamics of the Universe in this work.

III. ENTROPY PRODUCTION

Let us return to the main issue of this work - the question of the flatness of the Universe. The fact that the critical density ρ_{crit} “surfaced” by itself indicates that the flatness of the Universe is not a random setting, but a thermodynamic requirement of the holographic limit. An explanation of flatness will be presented here in terms of maximizing horizon entropy production for a universe with flat geometry (with critical density).

A. Study of the behavior of the entropy production function

Let’s formulate the problem: we need to show that entropy production with increasing horizon is maximal when this occurs in a flat universe. Let’s analyze the ratio of the change in entropy to the change in the system’s action that leads to this change.

We will consider only how the curvature itself (the contribution of curvature K, denoted Ω_k) increases the action. Consider the parameter $\theta(\Omega_k)$ corresponding to the ratio of the entropy increment S to the action increment of general relativity $\mathcal{S}_{GR}$

$$\theta(\Omega_k) = \frac{\Delta S}{\Delta \mathcal{S}_{GR}}. \tag{15}$$

We consider the following aspects to analyze the behavior of $\theta(\Omega_k)$:

1. The excess action of three-dimensional geometry is proportional to the deformation energy of the physical vacuum.
2. At the Planck scale, the space metric experiences constant random oscillations.
3. The quadratic form transforms the equations for small fluctuations into a system of independent harmonic oscillators.
4. The positive definiteness of this quadratic form guarantees the stability of the ground (vacuum) state of spacetime.

According to [24, 25], $\Delta\mathcal{S}_{GR}$ for small deviations from flatness in quantum geometrodynamics will include a change in the "flat" action $\Delta\mathcal{S}_{GR}(0)$ and a quadratic form associated with the vacuum deformation energy:

$$\Delta\mathcal{S}_{GR} \approx \Delta\mathcal{S}_{GR}(0) + \gamma\Omega_k^2 + O(3), \tag{16}$$

where γ is a positive value, since the flat state is the vacuum minimum. Then, up to third-order smallness,

$$\theta(\Omega_k) = \frac{\Delta S}{\Delta\mathcal{S}_{GR}(0) + \gamma\Omega_k^2}. \tag{17}$$

Let us take the derivative of $\theta(\Omega_k)$ with respect to Ω_k, taking into account that neither ΔS nor $\Delta\mathcal{S}_{GR}(0)$ depend on the curvature:

$$\frac{d\theta(\Omega_k)}{d\Omega_k} = -\frac{2\gamma\Delta S\Omega_k}{(\Delta\mathcal{S}_{GR}(0) + \gamma\Omega_k^2)^2}. \tag{18}$$

To find the extremum, we set the expression (18) equal to 0:

$$-\frac{2\gamma\Delta S\Omega_k}{(\Delta\mathcal{S}_{GR}(0) + \gamma\Omega_k^2)^2} = 0. \tag{19}$$

Obviously, the only solution to (19) is $\Omega_k = 0$, i.e., the case of a flat space. Analysis shows that function (18) is decreasing; as a consequence, the second derivative is negative, and therefore this extremum is a maximum. The general conclusion from the analysis is that the entropy production function $\theta(\Omega_k)$ reaches its maximum value in a flat universe.

B. Determination of the type of the entropy production coefficient $\theta(t)$

We will consider a scenario of the expansion of the Universe from an initial state with horizon R_1 over the Planck time T_{Pl} and calculate the ratio of the entropy increment to the action expended. We will start from the formulas for flat space ($k = 0$). We will also assume that T_{Pl} is much shorter than the time of the Universe’s existence at the initial moment.

The total increase in the entropy of the horizon over time T_{Pl} will be:

$$\Delta S = \frac{3\pi k_B c^5}{\hbar G H_1}\left(1+\frac{P_1}{\rho_1 c^2}\right)T_{Pl}, \tag{20}$$

where P_1 is the pressure, ρ_1 is the energy density. From here on, the index 1 refers to the start of integration.

The total Einstein-Hilbert action $\mathcal{S}_{tot}$ below the Hubble horizon is determined by two terms: the volume term $\mathcal{S}_{vol}$ and the surface Gibbons-Hawking-York term $\mathcal{S}_{GHY}$

$$\mathcal{S}_{tot} = \mathcal{S}_{vol} + \mathcal{S}_{GHY} \tag{21}$$

Volumetric component:

$$\mathcal{S}_{vol} = -\frac{c^5}{2G}\int dt \frac{1}{H(t)} \tag{22}$$

Surface part:

$$\mathcal{S}_{GHY} = -\frac{3c^5}{2G}\int dt \frac{1}{H(t)} \tag{23}$$

The total action for the Universe $\mathcal{S}_{tot}$ takes the form:

$$\mathcal{S}_{tot} = -\frac{2c^5}{G}\int dt \frac{1}{H(t)} \tag{24}$$

and the increment of this action over time T_{Pl}, respectively,

$$\Delta\mathcal{S}_{tot} = \frac{2c^5}{GH_1}T_{Pl} \tag{25}$$

We calculate the entropy production coefficient θ by dividing ΔS by $\Delta\mathcal{S}_{tot}$. After reductions and simplifications, we obtain

$$\theta = \frac{3\pi}{2}\frac{k_B}{\hbar}\left(1+\frac{P_1}{\rho_1 c^2}\right) \tag{26}$$

As expected, the expression includes Boltzmann's constant k_B and Planck's reduced constant $\hbar$ (the "quantum" entropy and "quantum" action). The dimensionless coefficient in parentheses is the parameter of the equation of state $w(t)$. The parentheses can be rewritten, generalizing to an arbitrary moment in time t (keeping in mind, of course, the initial assumptions and limitations). Accordingly, the entropy production coefficient becomes a function of time $\theta(t)$:

$$\theta(t) = \frac{3\pi}{2}\frac{k_B}{\hbar}\left(1+w(t)\right) \tag{27}$$

As a result, we obtained a pure dimensionless response of the geometry to the equation of state of the medium. A distinctive feature of this expression is its dependence *only* on the equation of state parameter $w(t)$.

IV. CONCLUSION

This paper examines the dependence of the cosmological horizon entropy on the density of a universe with arbitrary curvature. It is shown that the horizon entropy is related to the critical density, suggesting that the universe is flat. Furthermore, the product of the horizon entropy and the critical density is a constant and is determined only by a set of fundamental constants. The calculated results are compared with one of the conclusions of the holographic dark energy (HDE) model, demonstrating their identity. An entropy mechanism is proposed that enables the possibility and ability to maintain the Universe's density equal to the critical value. A possible implementation of this mechanism is analyzed and discussed: maximizing entropy production during the expansion of a flat Universe compared to a Universe with curvature. An expression for the entropy production coefficient is derived through a parameter of the Universe's equation of state.

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